Stationary dark energy with a baryon-dominated era: solving the coincidence problem with a linear coupling

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We show that all cosmological models with an accelerated stationary global attractor reduce asymptotically to a dark energy field with an exponential potential coupled linearly to a perfect fluid dark matter. In such models the abundance of the dark components reaches a stationary value and therefore the problem of their present coincidence is solved. The requirement of a vanishing coupling of the baryons in order to pass local gravity experiments induces the existence of an intermediate baryon-dominated era. We discuss in detail the properties of these models and show that to accomodate standard nucleosynthesis they cannot produce a microwave background consistent with observations. We conclude that, among stationary models, only a time-dependent coupling or equation of state might provide a realistic cosmology.

I. INTRODUCTION

The recent observations of an accelerating expansion [1], together with new cosmic microwave, large-scale structure and lensing data, give a strong indication that the universe fluid is composed of at least four different components: relativistic matter ($\leq$0.01%), baryons (few per cent), dark matter ($\sim$30%) and dark energy ($\sim$70%). The present amount of these components raises some deep questions: Why are the dark matter and dark energy, which supposedly have a different scaling with time, almost equal right now [2]? Why are the baryons strongly suppressed with respect to dark matter? Why is the coincidence between the dark components relatively close to the equivalence between matter and radiation (as pointed out by [3])?

The problem of the present coincidence between dark energy and dark matter could be simply solved if the two fluids have the same scaling with time (let us call a system of fluids ρ_1, ρ_2 with identical scaling a "stationary" model, since $d(\rho_1/\rho_2)/dt = 0$). It is interesting to remark that such a scaling can soon be observationally tested [4]. It is well known that, assuming an exponential potential for the scalar field representing the dark energy, the field scales as the dominant component [5, 6, 7], but this does not produce acceleration. The question therefore arises of which is the most general system of dark matter (modeled as a perfect fluid) and dark energy (a scalar field) that allows an *accelerated stationary global attractor*. In this paper, following the arguments of ref. [8], we show that all models which contain an accelerated stationary global attractor reduce asymptotically to a system characterized by an exponential potential *and* a linear coupling between the two components. Systems of such a kind have been already analyzed first in refs. [9, 10] and successively by many authors [11, 12, 13, 14, 15], while their perturbations have been studied in ref. [16]. In order to pass local gravity experiments [17], we argue that it is necessary to break the universality of the coupling, leaving the baryons uncoupled or weakly coupled (see e.g. [11, 17, 18, 19, 20]). These assumptions completely define our cosmology.

The consequences of a linear coupling between the dark components are manifold. First, as already remarked, this explains the cosmic coincidence and the acceleration. Second, the accelerated regime explains the decay of the baryons with respect to the dark components. Third, the near coincidence of the equivalence between the luminous components (baryons and radiation) and the beginning of the dark era (dark equivalence from now) is automatically enforced. Fourth, the baryons are the dominant component between the two equivalence times, producing a decelerated epoch in which gravitational instability is effective. Fifth, the present status of the universe is independent of the initial conditions, being on a global attractor. In addition, it is to be noticed that the model requires only constants of order unity in Planck units, and that they are all fixed by the observations of the present energy densities and the acceleration.

Nothwithstanding these intriguing features, we show that the model we present here fails in satisfying at the same time the nucleosynthesis requirements together with producing an acceptable cosmic microwave background (CMB) angular power spectrum. In fact if the conditions for a standard nucleosynthesis are adopted, during the recent accelerated regime a fast growth of the perturbations is induced, which in turn causes an excessive integrated Sachs-Wolfe (ISW) effect, in contrast with the observations. Nevertheless, we think that the dynamics we discuss is interesting on its own.

The conclusion we draw is that only models with a time-dependent coupling or a potential more complicated that an exponential may contain an accelerated global attractor and at the same time be compatible with nucleosynthesis. A solution along these lines has been proposed in [21] where a non-linear modulation of the coupling allows nucleosynthesis to happen and structure to form in a regime of weak coupling, while the acceleration is produced in the subsequent (present) regime of strong coupling. In ref. [22] it has been shown that a similar mechanism may be realized in superstring theories.

II. HOMOGENEOUS SOLUTIONS

Let us show first why all two-fluid systems with a stationary accelerated attractor reduce asymptotically to the one investigated below. Let us write a generic coupled two-fluid systems with equations of state $p_x = (w_x - 1)\rho_x$ in a flat Friedmann metric as

$$\dot{\rho}_c + 3Hw_c\rho_c = \delta, \quad (1)$$
$$\dot{\rho}_\phi + 3Hw_\phi\rho_\phi = -\delta, \quad (2)$$

where the subscript c stands for cold dark matter (CDM) and the subscript ϕ for a scalar field. The Friedmann equation is

$$3H^2 = \kappa^2 (\rho_\phi + \rho_c),$$

where $\kappa^2 = 8\pi$ and $G = c = 1$. As shown in ref. [8], the stationary condition $d(\rho_c/\rho_\phi)dt = 0$, that is $\rho_\phi = A\rho_c$, can be satisfied only if

$$\delta = \sqrt{3\rho_c}\kappa (w_c - w_\phi) \frac{A}{\sqrt{1+A}}\rho_c. \quad (3)$$

Putting $w_c = 1$ and observing that $A = \Omega_\phi/(1-\Omega_\phi)$, we have

$$\delta = \sqrt{\frac{3\kappa^2\Omega_\phi}{2}} \frac{\eta - 1}{\sqrt{1+\eta}} |\dot{\phi}|\rho_c, \quad (4)$$

where $\eta = 2U/\dot{\phi}^2$, the ratio of the potential to kinetic scalar field energy (notice that $w_\phi = 2/(1+\eta)$). The stationary solution is accelerated if

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = \kappa^2(1+A)\rho_c \left(\frac{1-w_\phi}{2}\Omega_\phi - \frac{1}{6}\right) > 0,$$

which, for $2 > w_\phi > 0$, can be realized only if $\Omega_\phi > 1/3$ (i.e. $A > 1/2$) and

$$\eta > \frac{3\Omega_\phi + 1}{3\Omega_\phi - 1} > 2. \quad (5)$$

So far we repeated the steps of ref. [8]. Now, let us consider the asymptotic behavior of η. If $\eta \to 0$, the kinetic energy dominates over the potential energy, and the asymptotic solution is not accelerated. If, on the other hand, $\eta \to \infty$, the potential energy dominates, and the solution will be identical to that of a cosmological constant. In fact, in this limit $\dot{\phi} = 0$ and $w_\phi = 0$ and Eq. (2) gives $\delta = 0$ where

$$\delta = \rho_c\sqrt{3\kappa^2 U(\phi)\Omega_\phi}, \quad (6)$$

which implies $\rho_c \to 0$. Therefore, barring oscillatory solutions, only if $\eta \to$ *const.* the scalar field behaves as stationary accelerated dark energy (clearly this case includes also that of dark energy as a perfect fluid). When η is constant, the coupling reduces to

$$\delta = \sqrt{2/3}\kappa\beta|\dot{\phi}|\rho_c, \quad (7)$$

with

$$\beta = \frac{3}{2}(\eta - 1)\sqrt{\frac{\Omega_\phi}{1+\eta}}, \quad (8)$$

which is the form we study below. Moreover, it is easy to show that η =const. implies an exponential potential $U = U_0 e^{-\sqrt{2/3}\mu\kappa\phi}$ where

$$\mu = \frac{3}{\sqrt{\Omega_\phi(1+\eta)}} \left[1 + \frac{1}{2}(\eta - 1)(1 - \Omega_\phi)\right]. \quad (9)$$

The conclusion is that a linear coupling and an exponential potential represents the only non-trivial asymptotic case of stationary accelerated dark energy. It is not difficult to see that this theorem extends also to a Brans-Dicke theory with an explicit coupling between matter components. As will be shown below, such a solution is also a global attractor in a certain region of the parameter space.

The condition for acceleration, $\eta > (3\Omega_\phi + 1)/(3\Omega_\phi - 1)$, together with $\Omega_\phi < 1$, imply in Eq. (8) that

$$\beta > \sqrt{3}/2. \quad (10)$$

This limit is much larger than allowed by local gravity experiments on baryons, which give at most $\beta < 0.01$ (see e.g. [9, 17]). Therefore, the theory must break the universality of the coupling and let the baryons be decoupled from dark energy. An immediate consequence of the species-dependent coupling, so far unnoticed, can be seen by observing that the energy density of the dark components scales as

$$\rho_\phi \sim \rho_c \sim a^{-3\frac{\mu}{\mu+\beta}}. \quad (11)$$

For any $\beta > 0$ the energy density decays slower than in the standard matter-dominated Friedmann universe. Therefore, any uncoupled (or weakly coupled) component, as the baryons, decays faster than the coupled ones (see also [14]).

The cosmology we study below is a more realistic version of the one above: we include in fact radiation and baryons, both of which are coupled to the dark components only through gravitation. Once baryons and radiation decay away, we recover the stationary accelerated attractor. The Einstein equations for our model have been already described in [11], in which a similar model (but on a different attractor, i.e., for different parameters) was studied (see also ref. [23]). Here we summarize their properties. The conservation equations for the field ϕ , cold dark matter, baryons (b), and radiation (γ), plus the Friedmann equation, are

$$\begin{aligned}
\ddot{\phi} + 3H\dot{\phi} + U_{,\phi} &= -\sqrt{2/3}\kappa\beta\rho_c, \\
\dot{\rho}_c + 3H\rho_c &= \sqrt{2/3}\kappa\beta\rho_c\dot{\phi}, \\
\dot{\rho}_b + 3H\rho_b &= 0, \\
\dot{\rho}_\gamma + 4H\rho_\gamma &= 0, \\
3H^2 &= \kappa^2 (\rho_c + \rho_b + \rho_\gamma + \rho_\phi),
\end{aligned} \quad (12)$$

where $H=\dot{a}/a$ and $U(\phi)=U_0 e^{-\sqrt{2/3}\mu\kappa\phi}$ (we put $\dot{\phi}$ instead of $|\dot{\phi}|$ for generality). The coupling β can be seen as the relative strength of the dark matter-dark energy interaction with respect to the gravitational force. The only parameters of our model are β and μ (the constant U_0 can always be rescaled away by a redefinition of ϕ). For $\beta=\mu=0$ we reduce to the standard cosmological constant case, while for $\beta=0$ we recover the Ferreira & Joyce model of [6]. As shown in ref. [25], the coupling we assume here can be derived by a conformal transformation of a Brans-Dicke model, which automatically leaves the radiation uncoupled. To decouple the baryons one needs to consider a two-metric Brans-Dicke Lagrangian as proposed in [17]. Additional theoretical motivations for this kind of coupling have been put forward in ref. [14] and for coupled dark energy in general in ref. [24].

The system (12) is best studied in the new variables [11, 26] $x=\frac{\kappa}{H}\frac{\dot{\phi}}{\sqrt{6}}$, $y=\frac{\kappa}{H}\sqrt{U/3}$, $z=\frac{\kappa}{H}\sqrt{\rho_\gamma/3}$ and $u=\frac{\kappa}{H}\sqrt{\rho_b/3}$ and the time variable $\alpha=\log a$. Then we obtain

$$
\begin{aligned}
x' &= (z'/z-1)\,x-\mu y^2+\beta(1-x^2-y^2-z^2-u^2),\\
y' &= \mu xy+y\,(2+z'/z)\,,\\
u' &= -3/2u+u\,(2+z'/z)\,,\\
z' &= -z\left(1-3x^2+3y^2-z^2\right)/2,
\end{aligned}
\tag{13}
$$

where the prime denotes derivation with respect to α. The CDM energy density parameter is obviously $\Omega_c=1-x^2-y^2-z^2-u^2$ while we also have $\Omega_\phi=x^2+y^2$, $\Omega_\gamma=z^2$ and $\Omega_b=u^2$. The system is subject to the condition $x^2+y^2+z^2+u^2\le 1$.

The critical points of system (13) are listed in Tab. I, where p is the scale factor exponent, $a\sim\tau^{p/1-p}=t^p$, where $g\equiv 4\beta^2+4\beta\mu+18$, and where we used the subscripts b,c,r to denote the existence of baryons, matter or radiation, respectively, beside dark energy. In Tab. II we report the conditions of existence and stability of the critical points, denoting $\mu_+=(-\beta+\sqrt{18+\beta^2})/2$ and $\mu_0=-\beta-\frac{9}{2\beta}$.

In Fig. 1 we display the parameter space of the model, indicating for any choice of the parameters which point is a global attractor (notice that there is complete symmetry under $\beta\to-\beta$ and $\mu\to-\mu$). As in [11], in which the baryons have been included only as a perturbation, there exists one and only one global attractor for any choice of the parameters. The explicit inclusion of the baryons induces here two new critical points (b_b and f_b); moreover, contrary to [11], all the critical points with non-vanishing radiation are always unstable.

From now on, we focus our attention on those parameters for which the global attractor is b_c, the only critical point that may be stationary and accelerated. In Fig. 1 we show as a grey region the parameters for which this attractor is accelerated. When b_c is the global attractor, the system goes through three phases:

a) the radiation dominated era (the saddle b_r);
b) the baryon dominated era (the saddle b_b);
c) the dark energy era (the global attractor b_c).

The dynamics of the model is represented in Fig. 2 (trend of $\Omega_{c,b,\gamma,\phi}$) and in Fig. 3 (w_{eff}). During the various phases, the scalar field is always proportional to the dominant component, just as in the uncoupled model of ref. [6]. The three eras are clearly visible: first, the energy density is dominated by the radiation, with a constant contribution from the scalar field and a vanishing one from dark matter; then, the baryons overtake the radiation, and the scalar field scale accordingly; finally, the system falls on the final stationary accelerated attractor, where dark matter and dark energy share the energy density and the baryons decay away. The two parameters β and μ are uniquely fixed by the observed amount of Ω_c and by the present acceleration parameter (or equivalently by w_{eff}). For instance, $\Omega_{c0}=0.30$ and $w_{eff}=0.33$ gives $\mu=8$ and $\beta=16$, values which have been used in Fig. 2 and Fig. 3. With this value of μ we have during radiation $\Omega_\phi=6/\mu^2\simeq 0.09$, compatible with the nucleosynthesis constraints (see e.g. [6, 27]). To be more conservative, values of μ bigger than 11.5 would satisfy the requirement of having $\Omega_\phi<0.045$ during nucleosynthesis as suggested in [28] but the situation would be qualitatively similar. Once β , μ and the present baryon and radiation abundances are fixed, the model is completely determined, and the ratio of dark matter to dark energy is independent of the initial conditions.

The radiation equivalence occurs at a redshift z_{eq} given by $(1+z_{eq})=\Omega_{b0}/\Omega_{\gamma 0}\simeq 500$ for realistic values. The dark equivalence redshift z_{dark} can be found equating the baryon density and the dark energy density. From the conservation laws

$$
\rho_B\sim a^{-3},\rho_C\sim\rho_\phi\sim a^{-3\frac{\mu}{\mu+\beta}}\,, \tag{14}
$$

putting $r=\beta/\mu$ and approximating $\Omega_{\phi 0}\simeq\frac{r}{1+r}$ (valid for $\beta,\mu\gg 1$) it turns out that

$$
1+z_{dark}=\left[\frac{r}{\Omega_{b0}\,(1+r)}\right]^{\frac{1}{3}\left(1+\frac{1}{r}\right)}, \tag{15}
$$

For $r\simeq 2$, we obtain $z_{dark}\simeq 5$.

The three main observations we compare our model to, nucleosynthesis, structure formation and present acceleration, are produced in turn during the three eras. The background trajectory discussed above passes the nucleosynthesis constraint, yields the observed acceleration and explains the cosmic coincidence. However, as we show next, it gives an exceedingly large integrated Sachs-Wolfe effect.

III. PERTURBATIONS

Close to the critical points b_b and b_c the perturbations in the cold dark matter component (δ_c) and in the baryonic one (δ_b) grow as a power of the scale factor, that is $\delta_c=\delta_b/b=a^m$ (see ref. [34]). In this expression the

Table I: Critical points.

Point	x	y	z	u	Ω_ϕ	p	w_{eff}	w_ϕ
a	$-\frac{\mu}{3}$	$\sqrt{1-\frac{\mu^2}{9}}$	0	0	1	$\frac{3}{\mu^2}$	$\frac{2\mu^2}{9}$	$\frac{2\mu^2}{9}$
b_r	$-\frac{2}{\mu}$	$\frac{\sqrt{2}}{\|\mu\|}$	$\sqrt{1-\frac{6}{\mu^2}}$	0	$\frac{6}{\mu^2}$	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{4}{3}$
b_c	$-\frac{3}{2(\mu+\beta)}$	$\frac{\sqrt{g-9}}{2\|\mu+\beta\|}$	0	0	$\frac{g}{4(\beta+\mu)^2}$	$\frac{2}{3}\left(1+\frac{\beta}{\mu}\right)$	$\frac{\mu}{\mu+\beta}$	$\frac{18}{g}$
b_b	$-\frac{3}{2\mu}$	$\frac{3}{2\|\mu\|}$	0	$\sqrt{1-\frac{9}{2\mu^2}}$	$\frac{9}{2\mu^2}$	$\frac{2}{3}$	1	1
c_r	0	0	1	0	0	$\frac{1}{2}$	$\frac{4}{3}$	–
c_{rc}	$\frac{1}{2\beta}$	0	$\sqrt{1-\frac{3}{4\beta^2}}$	0	$\frac{1}{4\beta^2}$	$\frac{1}{2}$	$\frac{4}{3}$	2
c_c	$\frac{2}{3}\beta$	0	0	0	$\frac{4}{9}\beta^2$	$\frac{6}{4\beta^2+9}$	$1+\frac{4\beta^2}{9}$	2
d	-1	0	0	0	1	1/3	2	2
e	$+1$	0	0	0	1	1/3	2	2
f_b	0	0	0	1	0	2/3	1	–

Table II: Properties of the critical points.

Point	Existence	Stability	Acceleration
a	$\mu<3$	$\mu<\mu_+, \mu<\frac{3}{\sqrt{2}}$	$\mu<\sqrt{3}$
b_r	$\mu>\sqrt{6}$	unstable $\forall\mu,\beta$	never
b_c	$\|\mu+\beta\|>\frac{3}{2}, \mu<\mu_0$	$\beta>0, \mu>\mu_+$	$\mu<2\beta$
b_b	$\mu>\frac{3}{\sqrt{2}}$	$\beta<0, \mu>\frac{3}{\sqrt{2}}$	never
c_r	$\forall\mu,\beta$	unstable $\forall\mu,\beta$	never
c_{rc}	$\|\beta\|>\frac{\sqrt{3}}{2}$	unstable $\forall\mu,\beta$	never
c_c	$\|\beta\|<\frac{3}{2}$	unstable $\forall\mu,\beta$	never
d	$\forall\mu,\beta$	unstable $\forall\mu,\beta$	never
e	$\forall\mu,\beta$	unstable $\forall\mu,\beta$	never
f_b	$\forall\mu,\beta$	unstable $\forall\mu,\beta$	never

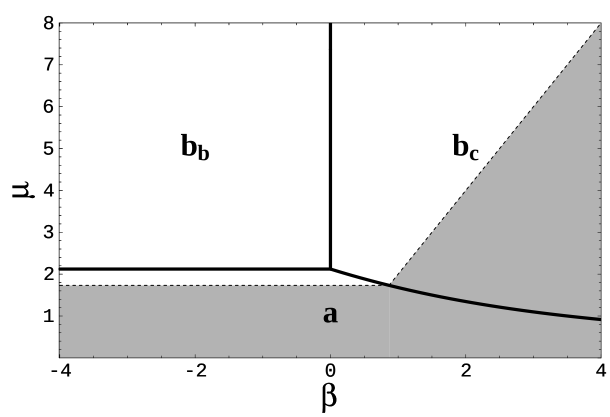

Figure 1: Parameter space. Each region is labelled by the point that is a global attractor there. Within the gray region the attractor is accelerated.

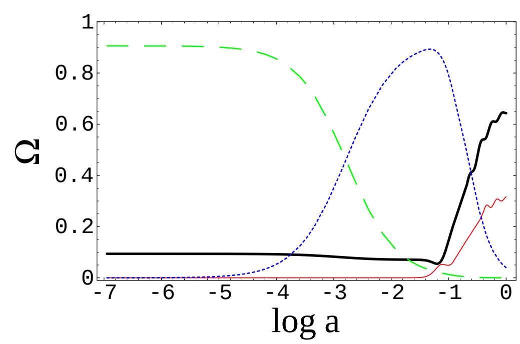

Figure 2: Trend of the radiation (dashed green line), dark energy (thick black line), dark matter (thin red line) and baryon (dotted blue line) density fractions, for $\mu=8$ and $\beta=16$ (here and in Figs. 3 and 6 the abscissa is $\log_{10}a$.)

baryon bias b and the growth exponent m depend only on the parameters μ and β. Considering only the dominating growing modes, during the baryonic phase the perturbations evolve asymptotically with the law (see for instance ref. [6])

$$m_1=\frac{1}{4}\left(-1+\sqrt{25-\frac{108}{\mu^2}}\right); \tag{16}$$

while in the last plateau the common growth exponent is

$$m_2=\frac{1}{4\left(\beta+\mu\right)}\left[-10\beta-\mu+\Delta\right]$$

where

$$\Delta^2=-108+44\beta\mu+32\beta^3\mu+25\mu^2+\beta^2\left(32\mu^2-44\right)$$

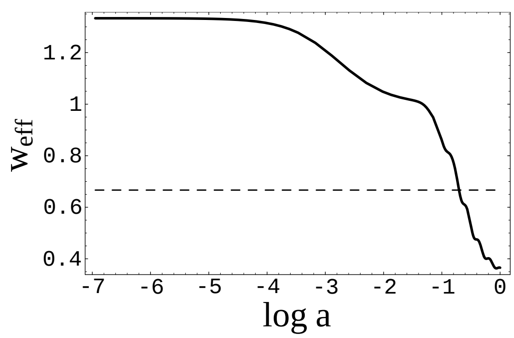

Figure 3: The effective equation of state $w_{eff} = 1 + p_{tot}/\rho_{tot}$ for the same parameters as in the previous figure. Below the dashed line the expansion is accelerated.

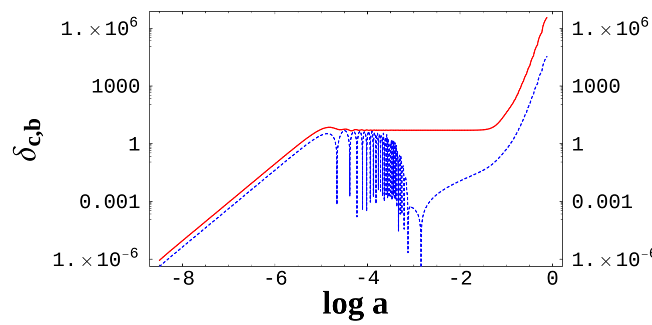

Figure 4: The evolution of a 10 Mpc/h perturbation for $\beta = 9.8$ and $\mu = 7$. The baryon fluctuations δ_b are represented by the dotted (blue) line, the CDM δ_c by the continuous (red) line.

In ref. [34] it is also reported how restrictions on the baryon bias would result in further constraints on the model but these will not be used here since experimental bias determinations still remain rather uncertain. The constraint provided by nucleosynthesis can surely be considered on firmer grounds.

The nucleosynthesis constraint $\mu \gtrsim 7$ (so that Ω_ϕ(1 MeV) $\lesssim$ 0.1) together with the limitation $0.6 < \Omega_{\phi 0} < 0.8$ implies a value of β comprised between 9.8 and 27.3. For this range of values the growth exponent in the last era is found to lie between 7.4 and 15.3 respectively. In Fig. 4 numerical evolutions of δ_c and δ_b using the full set of equations are shown in the case $\mu = 7$ and $\beta = 9.8$ for a fluctuation of wavelength 10 Mpc h^{-1} : the fast growth during the final stage shows up clearly. With such a conspicuous growth one expects a very large (late) ISW effect on the CMB, which in fact appears in the numerical integrations of the model of Fig. 5, produced using a version of CMBFAST modified for the dark energy. The angular power spectrum is forced, by the normalization procedure, to be highly suppressed at the lowest angular scales respect to the observed values. This determines the failure of the model investigated here. Even neglecting the nucleosynthesis constraint, the existence of a radiation era requires $\mu > \sqrt{6}$, a value that induces again an unacceptably large ISW effect.

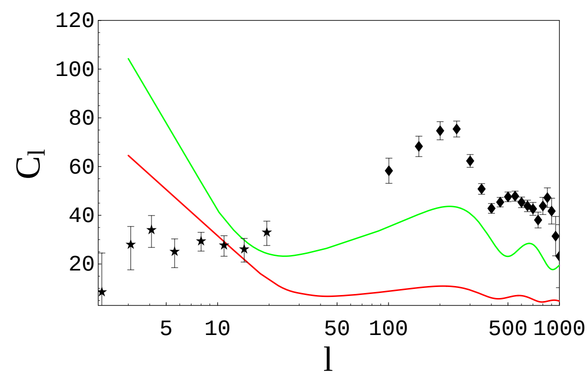

Figure 5: CMB power spectra C_ℓ for two set of parameters: $\beta = 4$,$\mu = 3.5$ (top curve) and $\beta = 3.3$,$\mu = 4.2$ (bottom curve), compared with observational data from Boomerang [30] and COBE [31]. The other input values for CMBFAST are $h = 0.8, \Omega_b = 0.04, \Omega_c = 0.3, n = 1$. The strong ISW effect at small multipoles is evident. Larger values of μ, as needed from the nucleosynthesis constraint, enhance the problem.

IV. CONCLUSIONS

Our coupled dark energy model provides a cosmological scenario with quite unusual features. The standard sequence of a dark matter era followed by a cosmological constant era, which forces to put the present universe on a unlikely transient, is here replaced by a baryonic era and a stationary dark era in which dark energy and dark matter share a constant fraction of the total density. Contrary to almost all models published so far, the present universe can be seen as already on the final global attractor (except that, luckily, some baryons are still around). The two new dimensionless constants introduced in our scenario, β and μ, are determined by the present dark matter energy density and by the present acceleration, and can be of order unity. All cosmologies with an accelerated stationary global attractor reduce asymptotically to the model discussed in this paper.

In this model the coincidence problem is immediately solved by setting β and μ to the same order of magnitude.

Regardless of the initial condition, the universe evolves to a stationary state with Ω_ϕ/Ω_c =const. *and* of order unity.

Moreover, this model explains also why the accelerated epoch occurs *just before* the present or, equivalently, why there are far less baryons than CDM. The reason is provided by Eq. (15): fixing $r = \beta/\mu$ of order unity and Ω_{b0} of the order of a few per cent, we have z_{dark} near unity, regardless of the initial conditions. That is, the fact that we observe a relatively small quantity of baryons around implies that the accelerated epoch is recent. Much more or much less baryons would push the beginning of the accelerated epoch far in the future or in the past.

The problem of the near coincidence between the radiation equivalence and the dark equivalence can be rephrased as why z_{eq} and z_{dark} are relatively close to each other. The answer is that is the end of the radiation era that triggers the onset of the baryon era, which in turn lasts for a relatively short time because the system is heading toward the global attractor represented by the dark era.

Despite these positive features, the model as it stands cannot explain our universe, since baryons and CDM fluctuations grow excessively during the last accelerated phase if it is considered that a standard nucleosynthesis has taken place earlier. This causes a CMB angular power spectrum suppressed at the lowest angular scales respect to the observational results. Thus we are forced to conclude that, for the universe to fall on the stationary attractor, a non-linear modulation in the coupling (as in ref. [21]), and/or a potential that reduces only asymptotically to a pure exponential, is needed.

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